Prediction of Deformation in the Potassium-Arrested Canine Left Ventricle Using the Finite Element Model

Toshio NAKAMURA, Hiroyuki ABE*, Koichi KASAHARA**, Motonao TANAKA***, Teruo KIMURA***, Masakichi MOTOMIYA*** and Shigeru ARAI****

Department of Internal Medicine 1, Tokai University School of Medicine.
*Faculty of Engineering, Tohoku University.
**Mitsui Engineering and Shipbuilding Co.
***Department of Medicine, Research Institute for Chest Diseases and Cancer, Tohoku University.
****Department of Pathology 1, Yamagata University School of Medicine.
(Received June 25, 1986)

A finite element model for the diastolic canine left ventricle based on large deformation theory has been developed. The myocardial stress-strain relation determined by the diastolic pressure-volume relation was used in this study. The ventricle was assumed as a homogeneous, isotropic, thick-walled solid of revolution with variable thickness. The predicted shapes based on the model were in close agreement with the shapes recorded by two-dimensional echocardiography. The stress calculated at the subendocardium was considerably larger as compared to that at the subepicardium. For example, at an intraventricular pressure of 10 cmH2O, the stress at the subendocardium was 80 g/cm2 and about four times larger than that at the subepicardium.

(Key Words: finite element model, large deformation theory, cardiac mechanics, wall stress, wall strain, myocardial stress-strain relation)

INTRODUCTION

Stress and strain distribution within the left ventricular wall provides us with one of the important informations for the evaluation of the regional function of the ventricle. It is possible to measure the strain within the left ventricular wall experimentally by placing radio-opaque markers (16) or ultrasonic dimension gauges (25). On the other hand, the measurement of the stress has been a problem which has not yet been solved. Several investigators (2, 7, 13) attempted direct measurement of the stress within the left ventricle. However, their results have been challenged by some researchers (8, 14), form the methodological point of view.

Recent advances in the methods of measurement of the shape and pressure of the left ventricle has enabled us to obtain more accurate information concerning the ventricular pressure-volume (P-V) relation. There have been numerous theoretical studies (17, 29) on the estimation of stress within the left ventricular wall. In these studies, the configuration of the left ventricle was approximated by models, such as a thin spherical shell or a thick spherical shell (29). In most of these models, deformation analysis was done based on the linear elasticity theory.

However, as pointed out by Mirsky (15), some doubt has been cast on the accuracy of the deformation analysis of the left ventricle based on the linear elasticity theory. For example, Mirsky (14) demonstrated that the circumferential stress within the ventricular wall which was determined by the large deformation theory differed markedly from that determined by the linear elasticity theory.

Our previous study (19) which was based on the large deformation theory dealt with the uniaxial stress-strain relation of a unit myocardium derived from the analysis of left ventricular P-V curve of the potassium-arrested canine heart. In the above study, the left ven-
tricle was assumed to be a thick spherical shell. However, in order to make a more precise analysis, it is necessary to analyze deformation of the left ventricle using a model which is much closer to the actual shape, such as a finite element model.

This study was one of the first attempts to analyze deformation of the ventricle by using an axisymmetric finite element model with variable thickness, in which the myocardial stress-strain relation was represented by the equation based on the large deformation theory. We obtained the circumferential stress and strain at the inner, middle, and outer portions of the left ventricular wall at every 0.5 cmH2O of the left ventricular pressure. In addition, prediction was made of the deformation of the left ventricle, and the shape of the left ventricle was calculated at every pressure. In this study, we also tried to evaluate the accuracy of this deformation analysis by comparing the measured and predicted shapes of the left ventricle.

**METHOD**

**Animal Experiment**

A mongrel dog weighing 17.5Kg was anesthetized by an intravenous injection of pentobarbital (25 mg/Kg). The thorax was opened under artificial respiration. A catheter was inserted through the left subclavian artery and advanced to the proximal portion of the aorta. After ligature of the descending aorta, 100 ml of saline containing 20 mEq/L KCl was injected rapidly through the cannula and the heart was arrested. Thereafter, the heart was immediately excised and both atria were opened and the ventricular cavities were washed with saline.

The chordae tendineae were severed and a purse-string suture was placed around the mitral annulus. A rubber plug (diameter of 2.5cm) with two stainless tubes (length; 7cm, and internal diameter; 2mm) was fixed at the mitral annulus. One end of each tube was placed in the left ventricular (LV) cavity. The other end of one tube was connected to the pressure transducer (Hewlett-Packard 1011C). Saline was infused through the other tube. The trunk of the left main coronary artery was dissected carefully and a cotton tie was placed beneath it. A modified Eckstein's cannula was inserted from the subclavian artery. It was passed through the aorta to the common left coronary ostium and fixed by ligature of the cotton tie. The LV cavity was filled with saline, air was driven out carefully and the aorta was ligated at the supravalvular portion.

The excised heart was kept submerged to a box (50cm × 100cm × 50cm) filled with saline of 20°C; the long axis of the left ventricle was laid parallel to the water surface with the

![Image](https://via.placeholder.com/150)

**Fig. 1** View of screen showing two-dimentional echocardiographical image of left ventricle at the pressure of 10 cmH2O.
free wall of the left ventricle above and the septum below. Before recording the P-V relation, the left ventricle was filled with saline three times up to a pressure of 30 cmH₂O. Thereafter, 2.5ml saline each was infused into the LV cavity at intervals of 10 seconds and the diastolic P-V relation was recorded.

During this procedure, cross sections of the left ventricle along the long axis were echographed on two-dimensions at pressures of 0, 5 and 10cmH₂O. Figure 1 shows one of the typical recordings of the LV shape at a pressure of 10 cmH₂O. A 2.25 MHz transducer with a diameter of 30mm and focus depth of 10cm was used. At the end of experiments, the LV pressure was set at 0cmH₂O and the heart was fixed by perfusion of 100ml of 0.05M phosphate buffer (pH = 7.4) containing 2% formalin and 2% glutaraldehyde through the above cannula. Both atria were dissected along the atroventricular groove; the right free wall was cut according to Fulton's method (5); the LV weight and intraventricular volumes were determined. The left ventricle was cut sagitally, and the outer and inner borders of the sections were traced. The heart was fixed within 30 minutes after cardiac arrest.

Theory
Pressure-Volume Relation
We defined the following equation:

\[ \Delta V = V - V_0, \]

where \( \Delta V \) is the actual ventricular volume change, \( V \) is the LV volume at a pressure of \( P \), and \( V_0 \) is the LV volume at a pressure of 0 cmH₂O. The P-V relation obtained experimentally in the previous study (18) was expressed as follows,

\[ \Delta V = a - b \cdot \exp(-c \cdot P), \]

where \( a \), \( b \) and \( c \) were constants. The constants \( a \), \( b \) and \( c \) were determined by a computer using the program of a corrected logistic curve (18).

Large Deformation Analysis
A pseudo strain-energy function which characterizes the elastic behavior of the LV myocardium during diastole was introduced and was determined by the above LV P-V relation based on the large deformation theory. The strain-energy function represents the energy per unit volume which is stored in the deformed material.

In the following study, the LV wall was assumed to be constituted by a homogeneous, isotropic and incompressible material, and the shape of the left ventricle was assumed to be a thick spherical shell. The deformation considered here was quasi-static, so that the viscous component was neglected. \( (r, \theta, \phi) \) and \( (R, \theta, \phi) \) represented the systems of spherical polar coordinate introduced in the underformed \( (P = 0 \text{ cmH}_2\text{O}) \) and in the deformed ventricles, respectively. The internal and external radii of the underformed ventricle were denoted by \( r_1 \) and \( r_2 \), respectively, and the corresponding radii after deformation by \( R_1 \) and \( R_2 \), respectively.

Under the condition of incompressibility,

\[ r^3 - R^3 = r_1^3 - R_1^3 = r_2^3 - R_2^3. \] (3)

We defined the following equation :

\[ Q(R) = \frac{r}{R} = \left[ 1 + \frac{1}{R}(r_1^3 - R_1^3)^{\frac{1}{3}} \right]. \] (4)

The large deformation theory can accurately deal with the deformation of geometrically non-linear material irrespective of the size of strain. As regards materials with non-linear properties, however, the accuracy of the analysis depends on the form of the pseudo strain-energy function. In the present study, the energy function was assumed to be a function of strain invariants with unknown coefficients, and was determined numerically in such a way that the equilibrium equation was satisfied through the thickness.

The pseudo strain-energy function was expressed with reference to the experiment of Demirays (5) and to our previous works (19) as follows:

\[ W = \frac{h_1}{b_1} \left[ \exp(b_1(I_1 - 3)) - 1 \right] + h_2(I_2 - 3), \] (5)

where \( I_1 \) and \( I_2 \) are strain invariants, and \( b_1 \), \( h_1 \) and \( h_2 \) are unknown coefficients.

By definition,

\[ I_1 = Q^4 + \frac{2}{Q}, \quad I = \frac{1}{Q^4} + 2Q^2. \] (6)

The unknown coefficients were determined
by the equation of equilibrium in the thick spherical shell.

The equation of equilibrium for the thick-walled spherical shell is given by

\[ K(Q_2) = P_1. \]  

(7)

The \( K(Q) \) is expressed by the following equation,

\[ K(Q) = 4 \int_{Q_1}^{Q_2} \left\{ b_1(1 + Q^3) \exp\{b_1(I_1 - 3)\} ight\} 
+ h_2 \left( Q + \frac{1}{Q^2} \right) dQ, \]  

(8)

\[ Q_1 = \left( 1 + \frac{V}{V_o} \right)^{-\frac{1}{3}}, \]  

(9)

\[ Q_2 = \left[ 1 - \left( \frac{r_1}{r_2} \right)^3 \left( 1 - \frac{1}{Q_1^3} \right) \right]^{-\frac{1}{3}}, \]

\[ r_1 = \left( \frac{3V_o}{4\pi} \right)^{\frac{1}{3}}, \]  

\[ r_2 = \left( r_1^3 + \frac{3W_L}{4\pi \rho} \right)^{\frac{1}{3}}, \]

where \( P \) (\( P = 1.05 \text{ g/cm}^3 \)) is the density of the LV wall and \( W_L \) is the LV weight.

Given \( m \) points of measured \( P_{ij} \) (\( i = 1 \sim m \)), the following equation can be applicable to these points,

\[ K(Q_2) = K(b_1, h_1, h_2) = P_1. \]  

(11)

Therefore, \( b_1, h_1, \) and \( h_2 \) are settled by the following equation.

\[ \left| K_i(b_1, h_1, h_2) - P_{ij} \right|^2 = \min. \]  

(12)

**Finite Element Model**

The axisymmetric finite element model based on a finite deformation theory was used in this study. The formation essentially the same as the works by Oden and Key (20) and Oden (21). The shape of the left ventricle was assumed to be a shell of revolution with variable thickness, the rotating axis of which was the long axis of the left ventricle. Figure 2 show a cross sectional view of the proposed finite-element model for the left ventricle. The free-wall geometry of the potassium-arrested left ventricle at an intraventricular pressure of 0 cmH2O defined the geometry of the model. The model was constituted by 104 elements and 73 nodes. With this model, the LV P-V relation was calculated by changing the pressure by 0.5 cmH2O at each step and the deformed shapes of the left ventricle at each intraventricular pressure were determined. The LV cavity was approximated by a polyhedron with \( n \) nodes its volume was determined as follows:

\[ V = \frac{\pi}{3} \left| \sum_{i=1}^{n-1} (z_{i+1} - z_i) \left( r_{i+1}^2 + r_i \cdot r_{i+1} + r_i^2 \right) \right| \]

\[ = \frac{\pi}{3} \left| \sum_{i=1}^{n-1} (r_i \cdot z_{i+1} + z_i) (r_i + r_{i+1}) \right| \]

\[ + \left( r_n \cdot z_n - r_1^2 \cdot z_1 \right). \]  

(13)

The physical components of the circumferential strain (\( \varepsilon_{\theta} \)) and circumferential stress (\( \sigma_{\theta} \)) were introduced according to the following equation:

\[ r_{\theta} = \frac{1}{r} \cdot r_{33}, \quad \sigma_{\theta} = \lambda^2 \cdot r^2 \cdot \sigma_{33} \]  

(14)

where \( \lambda \) is the circumferential extension ratio, \( \gamma_{33} \) the circumferential Lagrangian strain tensor and \( \sigma_{33} \) the circumferential stress tensor in the convected coordinates after deformation.
**Correction of Left Ventricular Volume**

The shapes recorded at 5 and 10 cmH2O revealed that the diameter of the base of the left ventricle increased from 3.4 cm to 3.6 and 3.8 cm, respectively. Therefore, we had to correct the LV volume obtained by calculation when the diameter increased. The diameter R at the base of the model was assumed to increase at a constant rate of α along the r-axis. The LV volumes before and after correction were denoted by V and V', respectively:

\[ V = \pi \int_{0}^{H} R^2(z)dz. \]  

\[ V' = \pi \int_{0}^{H} [R(1 + \alpha)]^2dz \]

\[ = (1 + \alpha)^2 \cdot \pi \cdot \int_{0}^{H} R^2(z)dz \]

\[ = (1 + \alpha)^2 \cdot V. \]  

\[ \alpha = \frac{\Delta R}{R_0}. \]  

where H is the height of the left ventricle, Ro is the radius of the base at a pressure of 0 cmH2O, Δ R is the increment of the radius of the base.

**RESULTS**

The LV weight (WL) was 108.0g and the LV volume at a pressure of 0 cmH2O (V0) was 28.0 ml. As regards coefficient constants in the P-V relation (Equation 2), a = 95.1 ml, b = 96.3 ml and c = 0.382 cm2/kg. The coefficients of the pseudo strain-energy function (Equation 5) were as follows: b1 = 1.51, h1 = 3.29 × 10^{-3} kg/cm^2 and h2 = 0 kg/cm^2.

Figure 3 illustrates the predicted shapes of the left ventricle at pressures of 5 and 10 cmH2O, and the underformed shape of the ventricle. In Figures 4 and 5, shapes predicted at pressures of 5 and 10 cmH2O are compared with the corresponding ones, respectively, which were determined by the two-dimensional echocardiography. The predicted figures were in close agreement with the figures recorded at the outer borders of both ventricles. At the inner borders, however, there were some discrepancies between the prediction and the experiment.

The changes in circumferential strain (\(\varepsilon_\theta\)) and stress (\(\sigma_\theta\)) of the inner, middle and outer portions of the LV wall were calculated on an equatorial plane (Figs. 6 and 7). The elements 39 and 40 corresponded to the inner portions, the elements 41 and 42 to the middle portions, and the elements 43 and 44 to the outer portions (Fig. 2).

Both circumferential stress and strain increased from the epicardial to the endocardial surface in a similar manner, when compared...
Fig. 4  A comparison of predicted and observed free-wall geometry at the pressure of 5 cmH₂O.

Fig. 5  A comparison between prediction and experiment of the free-wall geometries of the canine left ventricle at the intraventricular pressure of 10 cmH₂O.

Fig. 6  Circumferential strains near the equatorial plane predicted by the finite element model. Refer to Fig. 2 for the locations of finite elements.

Fig. 7  Circumferential stress near the equatorial plane predicted by the same model. The numbers in this Figure are the same as those in the previous Figure.
at the same intraventricular pressure. When the intraventricular pressure increased, the rate of increase in stress and strain of the ventricular wall was largest at the endocardial portion, followed by the middle, and epicardial portions. As a result, the differences of stress and strain within the ventricular wall increased further with an increment of the intraventricular pressure. But the rate of increase of strain was smaller than that of stress.

**DISCUSSION**

The study demonstrates that the large deformation analysis with a finite element model is able to predict the diastolic deformation of the left ventricle with sufficient accuracy. In the previous study (19), we reported the deformation analysis of the potassium-arrested heart based on the large deformation theory and derived the myocardial stress-strain relation of the diastolic ventricular wall from the ventricular P-V relation. In the above study, the left ventricle was assumed to be a thick-walled spherical shell as a first order approximation. In this study, however, we used a model much closer to the actual left ventricle. In other words, the left ventricle was assumed to be an axisymmetric finite element model with variable thickness.

**Finite element model**

Recently, the finite element method has been introduced for the analysis of the stress and strain of the cardiovascular system. The finite element models, so far used in ventricular deformation studies are classified into the following two categories: axisymmetrical (9,10,11) and nonaxisymmetrical models (6,22,23,28).

Janz et al. studied on the deformation of the potassium-arrested rat heart, using an axisymmetrical model. In their earlier study (10), the myocardium was assumed to be a linear elastic continuum. However, in their continued study (11), the elastic behavior of the myocardium was determined based on the experimental results of the uniaxial stress-strain relation of the papillary muscle. All the nonlinear terms in the strain-displacement relation were included in the formula of the finite element analysis. In other words, their model incorporated both nonlinear elastic and nonlinear geometric effects. However, in order to obtain an accurate correspondence in shape between the prediction and the experiment, they had to postulate that the elasticity of the inner one-third of the ventricular wall muscle was different from that of the outer two-thirds.

We used the myocardial stress-strain relation of the ventricular wall which was derived from the diastolic P-V curve of the left ventricle, based on the large deformation theory. In contrast to the study by Janz et al. (11), our results indicated that the finite element analysis based on the large deformation theory predicted the deformation of the free-wall side of the left ventricle without assuming the heterogeneity of the ventricular wall muscle. In addition, the predicted shapes were in close agreement with the shapes obtained by echocardiography of the same ventricle.

The shape of the left ventricle can be divided roughly into that of the free-wall side and that of the septal side. The axisymmetrical model mainly deals with the deformation of the free-wall. However, several morphological differences have been noticed between the free wall and septum, such as the fiber orientations (26) and the curvatures of the radii. Moreover, because of the differences in diastolic pressure between both ventricles, the transmural pressure of the septal side is usually lower than that of the free-wall side. Therefore, ideally, the deformation analysis based on the non-axisymmetrical model is preferable.

Several investigators have already carried out the deformation analyses of the left ventricle using non-axisymmetrical models (6, 22, 23, 28). Most of them used two shells of revolution which were generated by rotating the left- and right-hand sides respectively of the left ventricle about its longitudinal axis. In all of these analyses, however, the linear elasticity of the myocardium was assumed. Therefore, the validity of the stress and strain obtained by calculation in these studies has been challenged by several investigators as discussed below.

**Large deformation analysis**

The necessity of the use of large deformation theory in the study of the deformation of the ventricle has been indicated by Mirsky (14), in order to obtain more accurate and quantitative data on the ventricular wall stress. He also demonstrated that the stress distribution within the ventricle, which was obtained based on the
large deformation theory, was considerably different from that obtained by the conventional infinitesimal theory.

Janz et al. (11) represented the myocardial elasticity by the passive elastic behavior of the rat papillary muscle observed under the uniaxial tension. However, recent studies (12, 24) on the uniaxial tests of the papillary muscle indicated that the change in compliance of the damaged portion, at which the muscle was fixed, exerted a profound influence on the total elasticity of the muscle. Moreover, Streeter et al. (26) and Tezuka (27) demonstrated that the fiber orientation within the LV wall was entirely different from that of the papillary muscle. Therefore, a question arises whether the elastic relation obtained with papillary muscle is applicable to the deformation of the whole left ventricle.

In this study, on the other hand, we used a constitutive relation of the ventricular wall muscle which was determined by the analysis based on the large deformation theory. Despite the fact the myocardial stress-strain relation obtained in the experiment represented the elastic behavior of the bulk ventricular wall, the free-wall geometries predicted by our model were in close agreement with observed free-wall geometries of the left ventricle.

Wall stress and strain

In the present study, we for the first time calculated the wall stress distribution through the ventricular wall with a finite element model in which the myocardial elasticity determined by the large deformation theory was used. In a similar study of Janz et al., they calculated the strain as shown in Figs. 6 and 7 of their paper (11), but they made on mention of the predicted stress. The present study demonstrated that the wall stress at the subendocardium was larger than that at the subepicardium. Our results were quantitatively similar to those of the Pao et al.’s study (23). However, it was found that the decrease of stress across the wall thickness was of larger magnitude in our studies than that in their studies. Our results on the wall strain were similar to those on the wall stress. However, the rate of increase in strain in association with the increase of the ventricular pressure was somewhat smaller than that of stress.

In addition, it was found that the rate of increase of wall stress in association with the increase of intraventricular pressure at the endocardium was considerably larger than that at the epicardium. This result indicates that, at a higher end-diastolic pressure, the wall stress at the endocardium has a very large value as compared to at the epicardial portion. These data substantiate the findings that the subendocardium is more susceptible to ischemia than the subepicardium at the higher preload (1). However, these higher stresses obtained at the subendocardial portion may be reduced up to a certain point by the existence of trabeculae carneae at the internal surface of the left ventricle, because the trabeculae carneae increased the internal surface area of the left ventricle.

Limitations

One of the difficult problems related to the theoretical approach to the prediction of stress is that the comparison of the calculated results with direct measurements of the ventricular wall stress is as yet almost impossible. Recently, Huisman et al. (8) reviewed some of the limitations and problems associated with the direct measurements of stress of the ventricular wall. They indicated that current techniques do not allow direct and reliable quantification of wall stress, so that attempts at accurate calculation of the myocardial wall stress are more advantageous than the measurements of wall stress. On the other hand, Yin (29) suggested that, until accurate and reliable methods to measure wall stresses are developed, use of one of simpler models would probably suffice, because the predicted values cannot be verified by current means.

However, because of marked improvement of measuring instruments, it has become possible to record the deformation and strain much more easily and accurately than before. Therefore, indirect confirmation of the validity of the analysis is possible by comparing shapes between prediction and experiment. In the present study, the LV volume was changed by 260% from the undeformed state as a result of 10 cmH2O increase of the intraventricular pressure. Despite such a large deformation of the ventricle, our study indicated that the finite deformation method is able to predict the change in shape of the ventricle accurately.
Therefore, our results presented an indirect evidence for the validity of the calculation of stress using the large deformation theory. This study was carried out under the presumption that the myocardium is homogeneous and isotropic. However, variations in fiber orientation, for example, may exert a substantial influence on the size of wall stress obtained by calculation. Therefore, we need further studies based on the model in which the effect of heterogeneity of the ventricular wall is incorporated.

In this study, the heart beat was arrested by an infusion of potassium solution and the left ventricular pressure-volume relation were recorded within 30 minutes after excised. In our previous study (18), we have examined that the pressure-volume relations of the excised heart remained unchanged for 30 minutes after the arrest by an infusion of potassium. The same result was obtained in the anoxic arrested hearts by the experiment of Diamond et al. (4). Therefore, we supposed that the myocardial diastolic stress-strain relation of this ventricle did not change during the measurement.

CONCLUSION

Deformation of the left ventricle can be predicted accurately by using a finite element model, in which the myocardial elasticity represented by an equation based on the large deformation theory is used. The calculated results demonstrated that the circumferential stress is higher at the subendocardial portion than that at the subepicardial portion and that their difference across the wall becomes larger when the intraventricular pressure increases.

ACKNOWLEDGMENTS

This work was in part supported by Grant-in-aid from Mochida Memorial Foundation for Medical and Pharmaceutical Research.

REFERENCES

11) Janz RF, Kubert BR, Moriarty TF, Grimm AF: Deformation of the diastolic left ventricle - II. Nonlinear geometric effects. J Biomech 7: 509–516, 1974


